An Analysis of 2D Target Positioning Accuracy for M-sequence UWB Radar System under Ideal Conditions

Michal AFTANAS, Jana ROVŇÁKOVÁ, Mária RIŠKOVÁ, Dušan KOCUR, Miloš DRUTAROVSKÝ

Department of Electronics and Multimedia Communications
Faculty of Electrical Engineering and Informatics, Technical University of Košice
Park Komenského 13, 04120 Košice, Slovak Republic

{Michal.Aftanas, Jana.Rovnakova, Maria.Riskova, Dusan.Kocur, Milos.Drutarovsky}@tuke.sk

Abstract. *This paper describes analysis of target positioning accuracy for Ultra WideBand (UWB) Maximum length binary sequence (M-sequence) radar system under ideal conditions in two dimensional space. In this case, it is shown that the target position estimation error is cased by quantization of Time Of Arrival (TOA) measured by the radar system. The appropriate statistical quantities are suggested for describing these errors. The properties of M-sequence UWB radar systems performing under ideal conditions are illustrated by a set of simulations. The results obtained in this paper can be applied as a base for the design of positioning system based on M-sequence UWB radar.*

Keywords

UWB radar, M-sequence, location error, location accuracy, positioning, TOA, trilateration.

1. Introduction

Wireless location technology can provide real time indoor and outdoor positioning and tracking for various applications. Some potential uses include locator beacons for emergency services and mobile inventory or personnel and asset tracking for increased safety and security [1]. It can be also advantageously applied in field of navigation, e.g. as intelligent transportation systems [2], [3], etc.

Location systems can be classified into two broad categories: triangulation and trilateration systems. Triangulation systems use measurements of the bearings of targets from known points to find their locations. Trilateration (or multilateration) schemes find target positions by determining their ranges from known points (positions of RX and TX antennas). Range measurements are determined by estimating the Time Of Arrival (TOA) of the propagating signal between transmitter and receiver or by the Time Difference Of Arrival ($TDOA$) of the signal between two receivers [4].

The localization techniques allow two different positioning approaches: an active and a passive approach. The active method presumes that the target "cooperates" i.e. actively interacts with the positioning devices. In order to cover this proactive role, the target has to contain also a transmitting or receiving unit and it collaborates in the ranging measurements. The passive approach in contrast is based on a standalone positioning system exploiting "simple" wave scattering by the (unaware) target as in the case of a radar system [5].

This paper is intent on investigation of target positioning accuracy for trilateration location systems, which use TOA measurements and passive positioning approach. Such systems are represented e.g. by M-sequence UWB radar. The basic theory of M-sequences can be found e.g. in [6] and [7]. The M-sequence radar is characterized by a high spatial resolution and good penetration in materials, so one of its applications is detection and localization people hidden behind obstacles, e. g. in hostage scenarios, or detecting people buried during the disaster (avalanche, earthquake, etc). More about its architecture and performance properties can be found in [8] and [9].

The aim of this paper is to analyse target position accuracy in Two-Dimensional space (2D) in dependance on arrangement and number of antennas (one TX and two or three RX) in connection with true position of the target.

The structure of the paper is as follows. In Section 2, TOA positioning algorithm is described. It will be shown in Section 3, that the application of the M-sequence UWB radar for target positioning will result in target position error following from the fact that time quantities measured by M-sequence UWB radar system are expressed by an integer multiple of M-sequence chip interval (T_c). Following this idea, a set of statistical quantities describing target position estimation error will be proposed in the Section 4. Then, in the Section 5, computer simulations intend on evaluation of target positioning by the M-sequence UWB radar under ideal conditions will be presented. Finally, some conclusions from the presented work will be drawn in Section 6.

2. 2D TOA Positioning Algorithm

The problem description of the simplest case of 2D positioning based on TOA method is illustrated in Fig. 1. The aim is to determine passively, only by electromagnetic waves reflected from the target, the target location by using two receive antennas.

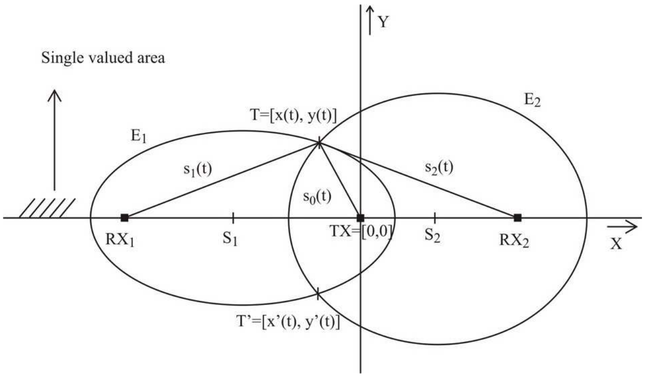

Fig. 1. 2D positioning with two receive antennas.

The most straightforward way for target position estimation is to solve directly a set of simultaneous equations made up on TOA measurements. The TOA express times of electromagnetic waves propagation from the TX towards the target T and reflected from the target T towards each of the antennas RX_1 and RX_2, respectively. By multiplying this TOA_i with the light speed c, the ranges $\text{TX-T-RX}_i = s_0(t) + s_i(t)$ will be obtained. They are related to the main half-axes a_i of an ellipse around the TX – RX_i antenna pair (it means TX and RX_i present a foci of the ellipse) by

$$a_i(t) = \frac{cTOA_i(t)}{2} = \frac{s_0(t) + s_i(t)}{2}, \ for \ i = 1,2. \quad (1)$$

The length of the minor half-axes b_i can be calculated by the expression

$$b_i(t) = \sqrt{a_i^2(t) - e_i^2}, \ for \ i = 1,2 \quad (2)$$

where e_i is given as half of distance between TX and RX_i. The last parameters which are needed to know for expressing appropriate equations of the above discussed ellipses are co-ordinates of centers $S_i = [sx_i, sy_i]$. In the case of scenario according to Fig. 1, $S_1 = [-e_1, 0]$ and $S_2 = [e_2, 0]$. Then, the set of equations describing ellipses E_1 and E_2 can be written as follows:

$$E_1: \ \frac{[x(t) + e_1]^2}{a_1^2(t)} + \frac{y^2(t)}{b_1^2(t)} = 1 \quad (3)$$

$$E_2: \ \frac{[x(t) - e_2]^2}{a_2^2(t)} + \frac{y^2(t)}{b_2^2(t)} = 1 \quad (4)$$

where $[x(t), y(t)]$ are target co-ordinates. They are computed analytically as the intersection of the ellipses E_1 and E_2. Because of the ellipses are expressed by polynomials of the second order, there are two solutions for their intersections that are related to the upper and lower half plane. If it is assumed that target can be situated only in one of this half plane, ambiguity will be removed. Such area will be referred as single valued area (Fig. 1).

In the case, that TX antenna and all RX antennas do not lie on the same straight line, the described ellipses have to be rotated. Then, the final equations are more complicated but the ellipse intersections still can be straightforward computed. The single valued area is now smaller than half plane and is exactly bounded by main axes of the rotated ellipses.

3. M-sequence as a source of target position estimation error

It is assumed throughout this paper, that M-sequence UWB radar system performs under ideal conditions. It means that:

- There is only one ideally pinpoint (infinity small) target.
- Signals received by the radar antennas are represented by single delayed M-sequences (i.e. there are no multiple reflections from target and no additional noise).
- There are no additional reflections (e.g. from walls) and no direct waves between TX-RX_i.
- Input Analogue-to-Digital Converter (ADC) has infinite bandwidth.

Under these ideal conditions, the main error of target position evaluation is caused by M-sequence application. Because the ADC input has infinite bandwidth, due to quantization effect, TOA of delayed M-sequence is quantized in time. If chip interval is set to T_c, the ADC will perform with sampling period set to T_c. This will cause quantization error E_Q of TOA measurement satisfying condition

$$\left|E_Q\right| \le \frac{1}{2}T_c. \quad (5)$$

The TOA is obtained from received delayed M-sequences by using matched filtering. Due to quantization, $TOA \in \left\{0, T_c, 2T_c, \ldots, \left(2^n - 1\right)T_c\right\}$ where $\left(2^n - 1\right)T_c$ is M-sequence period [9], [10], i.e. TOA is expressed by an integer multiple of M-sequence chip interval. Since the target position in 2D-space is computed from two TOA, quantization of TOA will result in error of target position determining.

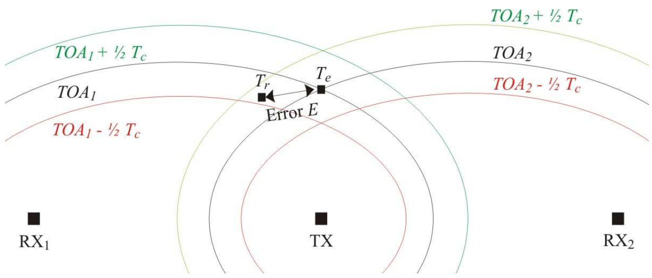

Fig. 2. *TOA* quantization effect illustration.

The above-described effect of TOA quantization is illustrated in Fig. 2. The position of each target having true position T_r in the area limited by $TOA_1 \pm 0.5T_c$ and $TOA_2 \pm 0.5T_c$, is estimated by the position T_e. It is an intersection of the ellipses given by TOA_1 and TOA_2. The ellipses due to $TOA_1 \pm 0.5T_c$ and $TOA_2 \pm 0.5T_c$ form the area borders where the true target position is locate. Then, the target position estimation error is given by the expression

$$E = |T_r - T_e| = \sqrt{(x_r - x_e)^2 + (y_r - y_e)^2} \tag{6}$$

where $T_r = [x_r, y_r]$ and $T_e = [x_e, y_e]$ are the true and estimated positions of target.

4. Statistical Description of Target Position Estimation Error

In order to examine statistical properties of target position estimation error due to TOA quantization effect, a sufficient number N of random true positions of target $T_{ri} = [x_{ri}, y_{ri}]$ is generated in 2D space with uniform probability distribution. Then, by using $T_{ri} = [x_{ri}, y_{ri}]$ and taking into account the TOA quantization effect, N estimate positions of target $T_{ei} = [x_{ei}, y_{ei}]$ are computed. By using T_{ri} and T_{ei}, a set of errors E_i is computed as follows

$$E_i = \sqrt{(x_{ri} - x_{ei})^2 + (y_{ri} - y_{ei})^2} \quad i = 1, 2, ..., N \tag{7}$$

and on of them is depicted in Fig. 2.

By using E_i, the maximum error of target position estimation can be computed by expression

$$E_{\max} = \max_{i=1,2...N} (E_i). \tag{8}$$

An average error E_a given by

$$E_a = \frac{\sum_{i=1}^{N} E_i}{N}. \tag{9}$$

is another statistical quantity proper for target position estimation error description.

In the next, the graph of an empirical Cumulative Distribution Function (CDF) of target position estimation error will be also used for M-sequence UWB radar performance description [5]. Here, CDF is defined by $CDF(E) = P(E \le e)$ where $P(E \le e)$ is a probability that target position estimation error E is less than or equal to e.

5. Computer Simulations

All simulations described in this section have been made in MATLAB under conditions given in the Section 3. The M-sequence UWB radar system with two and three RX antennas and one TX has been considered in our simulations. Numbers of chips of M-sequence and clock frequency $f_{clock} = \frac{1}{T_c}$ of M-sequence generator define so-called coverage area. It is the area where the target can be located by the used radar system. The coverage area is bordered by the conjunction of two ellipses given by $TOA = (2^n - 1)T_c$.

In our simulation M-sequence of the UWB radar with 511 chips ($n = 9$) and $f_{clock} = \frac{1}{T_c} = 9\,\text{GHz}$ have been used. Then, the maximum distance between TX – Target – RX is

$$d_{\max} = ct = 511cT_c = \frac{511c}{f_{clock}} = 17.033\text{m}. \tag{11}$$

All target positions analyzed in our simulations have been located in an area given by the conjunction of the single valued area and coverage area. In the next, this area will be referred as examinated area. Within this area, $N = 10^6$ of T_r are generated with uniform probability distribution. For each T_r, T_e is computed taking into account TOA quantization effect described in Section 3. For that purpose, the quantized value of TOA can be computed as follows

$$TOA_i = T_c round\left[\frac{s_0 + s_i}{cT_c}\right] \quad i = 1, 2. \tag{12}$$

Then, distance between T_e and T_r determines target position estimation error.

In the next, the following variations of basic scenario have been used in simulation:

- Two RX antennas and TX antenna are located on the same straight line (Scenario 1).
- Two RX antennas and TX antenna are tiled from axis X by an angle φ (Scenario 2).
- Two RX antennas and TX antenna are located on the same straight line and third RX antenna is located vertically over TX antenna (Scenario 3).

5.1 Scenario 1-2: Two Receive Antennas

The outline of the Scenario 1 is given in the Fig. 3.

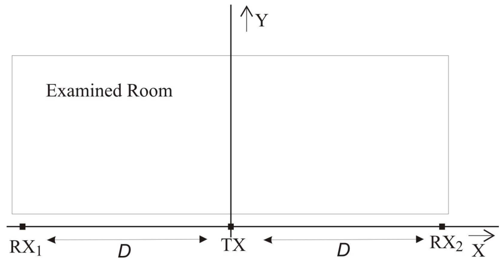

Fig. 3. Scenario 1. RX and TX antennas positions. *D* = 5m.

In this case RX_1 and RX_2 are distant from TX by distance D. In order to examine the target position estimation error properties, the distance D will be used as the parameter.

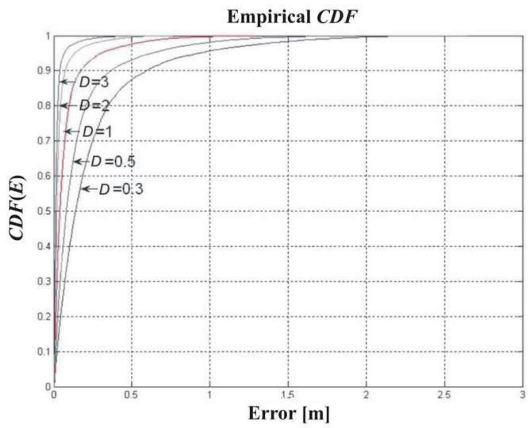

Fig. 4. Scenario 1. *CDF(E)*. $D \in <0.3m, 3m>$.

$CDF(E)$ corresponding to the Scenario 1 is given in the Fig. 4. It can be seen from this figure, that if the RX antennas are too close to TX antenna, the target position estimation error will be larger than they are farther, i.e. estimation error decreases with increasing of D.

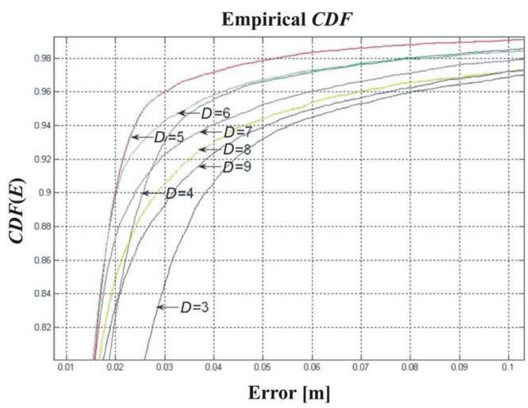

Fig. 5. Scenario 1. Zoomed $CDF(E)$. $D \in <3m, 9m>$.

A zoomed detail of $CDF(E)$ for $D \in <3m, 9m>$ is presented in the Fig. 5. It can be identified from this figure, that the best accuracy (the lowest error) is reached approximately, if D = 5m.

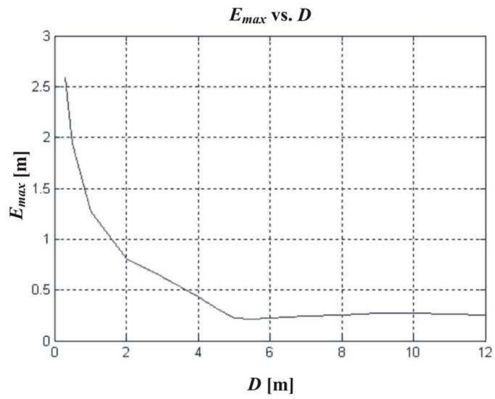

Fig. 6. Scenario 1. E_{max} vs. *D*. $D \in <0.3m, 12m>$.

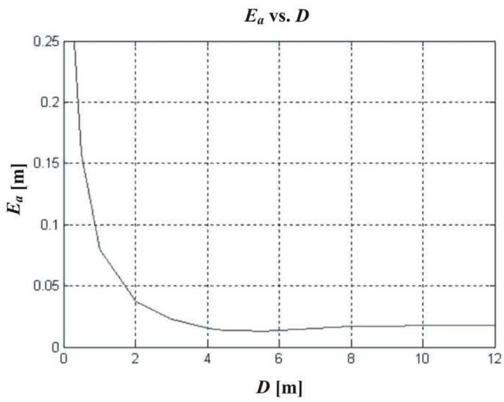

Fig. 7. Scenario 1. E_a vs. *D*. $D \in <0.3m, 12m>$.

The maximum and average error ($E_{\max}$, E_a) are given in the Fig. 6 and 7, respectively. It can be seen from this figures, that $E_{\max}$ and E_a are minimum approximately if $D \geq 5m$. If D is growing up the examinated area is decreasing really fast, e.g. between D = 5m and D = 12m examinated area decreases by 54 %. Then, for the Scenario 1, the optimum value of D is set to 5 m.

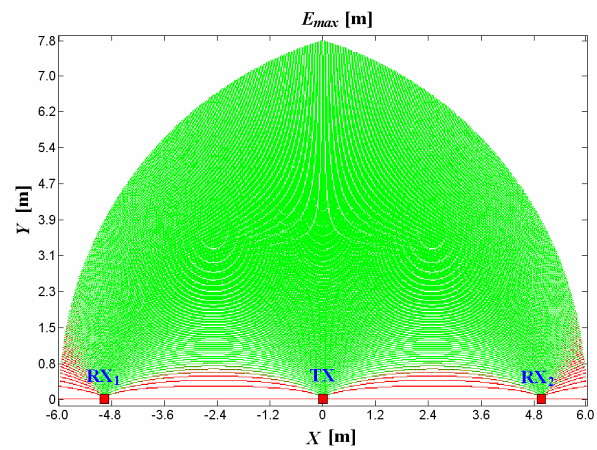

Fig. 8. Scenario 1. E_{max} distribution within examinated area. *D*= 5m. Green areas have lover error than red areas.

In Fig. 8, E_{max} distribution within examinated area is illustrated for D = 5m. Here, the green areas have lover error than red areas. It follows from this figure, that places localized between antennas possess the largest estimation errors. Other areas have nearly the same maximum error.

Of course all antennas do not have to be on one line.

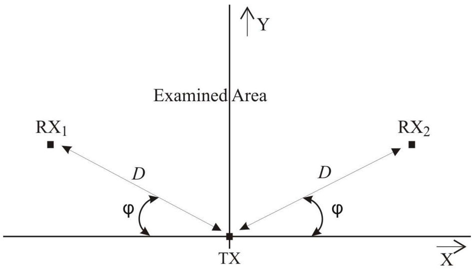

Fig. 9. Scenario 2. RX and TX antenna positions. D = 5m.

In Fig. 9 the second system that we choose for test is shown. RX antennas are tilted from axis X by an angle φ. In this case once again RX_1 and RX_2 are distant from TX by distance D, but now D is constantly 5 meters and parameter is the angle φ.

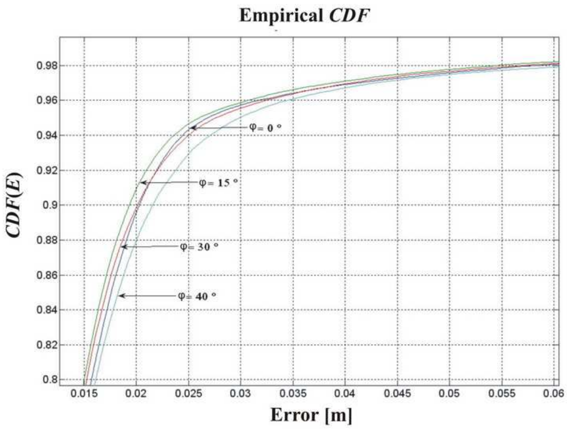

Fig. 10. Scenario 2. *CDF(E)*. $\varphi \in < 0^0, 40^0 >$.

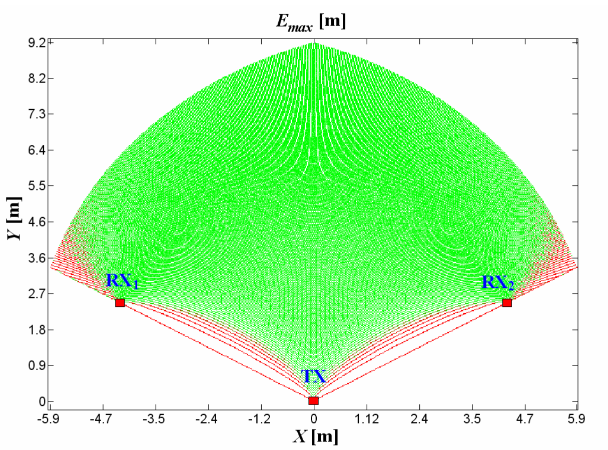

Fig. 11. Scenario 2. E_{max} distribution within examinated area. φ = 30 °. Green areas have lover error than red areas.

In Fig. 10 the detail of $CDF(E)$ is presented, where φ is applied as parameter. It can be seen from this figure, that target position estimation error almost does not depend on φ, therefore $CDF(E,\varphi) \approx CDF(E)$ for this scenario.

If the antennas are tilted, the examinated area is narrowed, as it can be seen in Fig. 11. The examinated area is smaller, but maximal distance from TX to target is larger. The largest error of target position estimation can be found on the line between TX and RX, in other parts of examined area, the errors are almost the same.

5.2 Scenario 3: Three Receive Antennas

The outline of the Scenario 3 is given in the Fig. 12. In this case, RX_1, RX_2 and TX antennas are on the same straight line and RX_3 is located vertically over TX antenna.

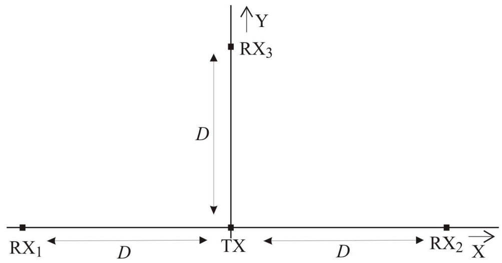

Fig. 12. Scenario 3. RX and TX antenna positions. D = 5m.

In this case, there are three TOA, from RX_1, RX_2 and RX_3. It is possible to compute the target position estimation from each two of them and in such a way to obtain three estimation of target position T_e. The resultant position is obtained by computing gravity centre corresponding to these three possible positions. In the Fig. 13, the $CDF(E)$ for this scenario can be found.

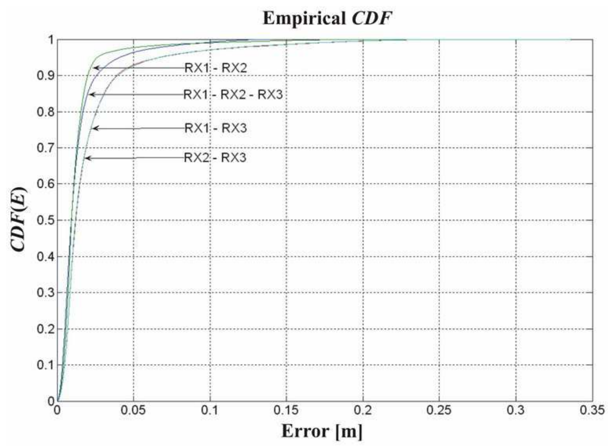

Fig. 13. Scenario 3. $CDF(E)$. D = 5m.

It can be seen that lowest position error is when a target position is computed from TOA_1 and TOA_2, but Tab.1 and Tab.2 show that maximal error E_{max} is lowest if the target position is computed by gravity centre from three positions. In some cases, it holds also for the average errors E_a .

D [m]	1	3	5	7	9	11
RX_1 - RX_2	1,278	0,628	0,210	0,238	0,269	0,256
RX_1 - RX_3	1,227	0,490	0,331	0,286	0,327	0,360
RX_2 - RX_3	1,224	0,460	0,346	0,294	0,331	0,360
RX_1 - RX_2 - RX_3	0,465	0,218	0,141	0,169	0,187	0,201

Tab. 1. Scenario 3. E_{max} [m].

D [m]	1	3	5	7	9	11
RX_1 - RX_2	7,87	2,34	1,29	1,51	1,63	1,58
RX_1 - RX_3	10,0	3,28	2,10	2,45	3,39	4,36
RX_2 - RX_3	9,65	3,19	2,08	2,43	3,44	4,39
RX_1 - RX_2 - RX_3	6,70	2,21	1,39	1,66	2,23	2,68

Tab. 2. Scenario 3. E_a [cm].

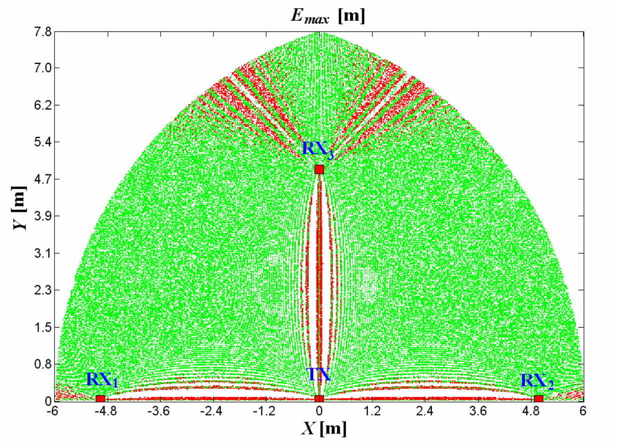

Fig. 14. Scenario 3. E_{max} distribution within examinated area. *D*=5m. Green areas have lover error than red areas.

$E_{\max}$ distribution within examinated area for the Scenario 3 is illustrated in the Fig. 14. It can be seen from this figure, that largest errors of target position estimation are localized along the straight lines between TX and all RX antennas. Other large errors are on line between RX antennas, but only behind antennas excluding area between them.

The presented results of simulations have been obtained for particular scenarios under ideal conditions. It is expected, that if these conditions are not fulfilled, additional sources of inaccuracy of target position estimation will have to be taken into consideration.

6. Conclusions

In this paper, a method for analysis of target positioning accuracy for M-sequence UWB radar system under ideal conditions has been proposed. It has been shown, that the target position estimation error is caused by quantization effects at *TOA* measurement by using M-sequences. By using this method, three different scenarios have been simulated. It has been found, that the largest target position estimation errors are located along the straight lines between TX and all RX antennas, and also on straight line between RX antennas, but in this case only behind antennas excluding area between them. The size of coverage area depends on arrangement of antennas. More RX antennas improve accuracy of UWB radar system.

The results described in this paper can be applied as a base for study of target position estimation by M-sequence UWB radar under more realistic scenarios. Beside, the proposed method can be applied in the process of sensor network design intent of detection, positioning and tracking of selected targets.

Acknowledgment

This work was supported by the Slovak Research and Development Agency under the contract No. LPP-0287-06.

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