Comparison of Basic Inversion Techniques for Through-Wall Imaging Using UWB Radar

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***Abstract*— In this paper, we consider the problem of detecting and localizing static objects in a room using though-wall Ultra-WideBand (UWB) radar SAR measurements. This paper investigates basic inversion (imaging) techniques which are used for interpreting the raw radar data in an understandable way. We also introduce a fast method for estimating the travel time between the antennas and the targets. The accuracy and computational complexity of the imaging techniques are analyzed and compared.**

I. INTRODUCTION

Through-wall imaging using radar offers a powerful tool for safety and security applications such as rescue operations, surveillance and hostage situations. In raw radar data, the information of targets is usually shown as hyperbolae. The aim of imaging techniques is to reconstruct the location and shape of the targets by focusing the hyperbolae to their true locations. Imaging methods can be divided into two categories: back-projection methods (they are also referred to as ray-based methods) and back-propagation methods (or wave-based methods). Back-projection methods make use of the direct ray path between the antennas and the targets and thus the electromagnetic wave theory is not taken into account. In contrast, back-propagation methods are based on the electromagnetic wave theory and can be subdivided into time-domain and frequency-domain methods. We note that this classification was not consistent in the literature.

The key point of these imaging methods is the estimation of the travel time between antennas and a possible target. A factor that must be taken into account when applying these imaging methods to through-wall data is the time delay due to propagation of the wave inside the wall. Ignoring this time delay can not only displace the reconstructed targets from their true positions but also defocus the target images. In this paper, we evaluate the performance of three basic imaging techniques: Synthetic Aperture Radar (SAR) (or diffraction stack migration), Kirchhoff migration and Stolt migration (or f-k migration). Moreover, we propose a fast method for calculating the travel time in through-wall scenarios.

In order to evaluate the imaging methods and compare their results, we introduce two parameters: the Signal to Clutter Ratio (SCR), which is defined as the ratio of the energy between the estimated regions of the objects and the clutter regions, and the relative positioning error (RPE), which is defined as the ratio between the error in the object position estimation and the ground truth. A good reconstruction method should have a large SCR and a small RPE.

The paper is organized as follows. Section II describes the measurement scenario used in this paper, system specification and data pre-processing steps. In Section III, the basic imaging techniques are briefly described. The estimation of the travel time is presented in Section IV. Section V is devoted to reconstruction results and analysis. Finally, some conclusions are drawn in Section VI.

II. MEASUREMENT SETUP AND DATA PRE-PROCESSING

A. Measurement Setup and System Specification

A simplified sketch of the measurement layout is shown in Fig. 1. The radar system was horizontally scanned parallel to the front brick wall (with a thickness of 18 cm) at a distance of 50 cm from the wall between position 1 and position 2. In this paper, we illustrate the performance of the imaging algorithms for the data of an aquarium of size 50 × 30 × 30 (cm) filled with water (Fig. 2(b)). The object was placed inside the room at position 3 which is at the center of the scanning range and 1 m far from the inside of the wall.

The UWB system used in this experiment is a pseudo random noise radar which consists of one transmitter and two receivers. The frequency band is between 800 MHz and

4.5 GHz. The antennas were fixed on a trolley in a vertical mask with the transmitter in the middle (see Fig. 2(a)). The distance between the center of the transmitting antenna and each of the receivers is 45 cm.

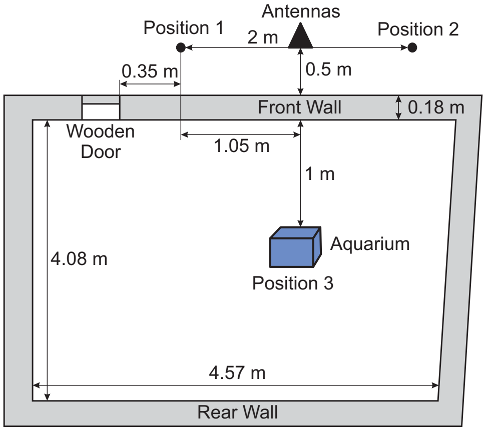


Fig. 1. The through-wall SAR measurement scenario.

(a) (b)

Fig. 2. The through-wall measurement scenario: (a) Radar system; (b) An aquarium filled with water.

B. Data Pre-processing

Before applying the imaging techniques, some pre-processing steps (time zero compensation, data interpolation, low-pass filtering, crosstalk removal and deconvolution) were applied to the measured raw data. The purpose of time zero compensation is to find, in the measured radar data, the exact time instant at which a signal is emitted from the transmitter and accordingly shift the measured data. Data interpolation is used to up-sample the measured data in both time and space variables. Low-pass filtering is applied to suppress any quantization noise. Deconvolution sharpens the received impulses by compensating for the radar transfer function. For more details on data pre-processing, the reader is referred to [6], [8], [10].

III. FORMULATION OF BASIC IMAGING TECHNIQUES

In this section, we summarize the formulation of three basic imaging techniques: SAR, Kirchhoff and Stolt migrations. Since the vertical resolution of the measured data is very low, we only consider a B-scan and focus it on the bisecting (horizontal) plane of a transmitter-receiver pair. On this plane, we define a Cartesian coordinate system of which the coordinates of a point are denoted by (x, z), where x is in the scanning (cross-range) direction and z represents the coordinate in the range direction. We denote by $\varphi(x, t)$ a measured B-scan with t being the time variable. A focused image in the object space is denoted by $I(x, z)$.

A. Synthetic Aperture Radar

SAR is a back-projection imaging method that was first developed for airborne radar applications, and later adapted to Ground Penetrating Radar (GPR) [4], [8]. The algorithm is based on the fact that an object contributes to the received waveforms with different time delays which depend on the distance between the antennas and the object. The object distribution can be estimated by focusing the received data using the following formula

$$I(x, z) = \int \varphi(x_a, \Delta t_{x_a}) dx_a, \tag{1}$$

where Δt_{x_a} is the travel time from the transmitter to the point $P(x, z)$ and back to the receiver. The integration is taken along the scanning path. Note that the travel time in free space is given by

$$\Delta t_{x_a} = [d(T_{x_a}, P) + d(R_{x_a}, P)]/c,$$

with $d(T_{x_a}, P)$ and $d(R_{x_a}, P)$ being the distances from the transmitter T_{x_a} to the point P and from P to the receiver R_{x_a}, respectively; c the light speed in free space.

The computational complexity of the SAR algorithm is of $O(N_x N_z N_a)$, where N_x and N_z are the number of points in the x- and z-directions in the object space at which the reconstruction is done, N_a is the number of A-scans. By subdividing the synthetic aperture into smaller ones, Yegulalp [9] showed that we can speed up by a factor of $\sqrt{N}$ if $N_x = N_z = N_a = N$.

B. Kirchhoff Migration

Kirchhoff migration is based on the Kirchhoff integral theorem representing the solution of the wave equation using Green's functions [3], [5]. Using the closed form of the Green's function for homogeneous media, the migrated field at a point $P(x, z)$ in a 2D homogeneous space for a monostatic radar system can be approximated by the following formula [3]

$$I(x, z) = \int \frac{\cos\theta}{\sqrt{\pi r v}} \partial_t^{1/2} \varphi(x_a, \Delta t_{x_a}) dx_a, \tag{2}$$

where θ is the angle between the z axis and the line joining the point $P(x, z)$ and the position of the antenna T_{x_a}; $\partial_t^{1/2}$ represents the half derivative with respect to time variable t. It is defined via frequency domain as

$$\mathcal{F}\partial_t^{1/2}\varphi(\omega) = (i\omega)^{1/2}\mathcal{F}\varphi(\omega).$$

Here $\mathcal{F}$ is the Fourier transform. r is the distance from the antenna to the point P and v is the wave propagation speed.

Note that (2) is formulated for homogeneous media. However, in this paper, it is also used in through-wall scenarios with an adaptation for the travel time Δt_{x_a} (Section IV).

Concerning the computational cost, the Kirchhoff migration is a bit more expensive than SAR as we need to calculate the derivative with respect to the time variable of the received signals. However, they are still in the same order as the cost for numerical integration dominates that of the fast Fourier transform used for calculating the derivative.

C. Stolt Migration

The F-k migration (also referred to as Stolt's migration) is a Fourier transform-based method developed from an exact solution to the wave equation. It is a solution to the migration problem for constant velocity in the propagation medium. It is a direct method that is the fastest known migration technique.

The use of F-k migration was first proposed by Stolt [7] for efficient processing of seismic data. This approach was later used also for GPR and UWB signal processing [2]. He formulated a migration solution that allows using the 2D FFT for computationally efficient processing. The basic formula of F-k migration is given by

$$I(x,z) = \iint \frac{v k_z}{\sqrt{k_x^2 + k_z^2}} \phi_0(k_x, f(k_z)) e^{2\pi i (k_x x - k_z z)} dk_x dk_z, \tag{3}$$

where $f = v\sqrt{k_x^2 + k_z^2}$ and

$$\phi_0(k_x, f) = \iint \varphi(x_a, t) e^{-2\pi i (k_x x_a - f t)} dx_a dt.$$

The computational complexity of the F-k migration is of $O(N_x N_z \log_2 N_a)$ which is known as the fastest available migration algorithm. However, certain distortion can appear as it requires the interpolation from (k_x, f) to (k_x, k_z). Hence there is a tradeoff between distortion and complexity.

IV. TRAVEL TIME ESTIMATION

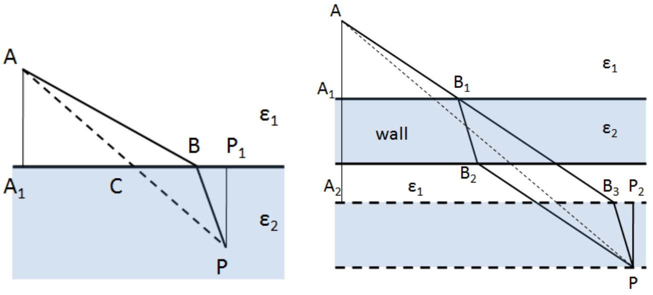
In through-wall scenarios, the time that a signal travels between the antennas and possible objects cannot be directly calculated unlike in homogeneous media. Instead, we have to take into account the refraction of the signal at the interface between layers. In general, the calculation of the travel time in multi-layered media is very time consuming. In this paper, we use a fast method for approximating it as in the case of 2-layered media. The idea was partly presented in [1].

For a 2-layered medium as shown in Fig. 3(a), the signal from the antenna A to the point P follows the path ABP. The inflection point B can be approximated as [4]

$$\overrightarrow{BP_1} = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \overrightarrow{CP_1}.$$

By simple manipulations, we can represent the inflection point B in terms of the positions of the antenna A, the point P and their projections on the interface as follows

$$\overrightarrow{BP_1} = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \frac{|PP_1| \overrightarrow{A_1P_1}}{|AA_1| + |PP_1|}. \tag{4}$$



(a) (b)

Fig. 3. Signal travel path in (a) 2-layered and (b) 3-layered media.

To calculate the travel path from antenna A to P in the through-wall scenario shown in Fig. 3(b), we imagine that the wall is "moved" toward the point P. From the figure we can see that the travel time following the path AB_1B_2P is the same as that of the path AB_3P. So it is enough to calculate the coordinates of B_3. Using (4) we arrive at

$$\overrightarrow{B_3P_2} = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \frac{|PP_2| \overrightarrow{A_2P_2}}{|AA_2| + |PP_2|}. \tag{5}$$

The travel time is then easily calculated from (5). It is used in SAR and Kirchhoff migration. However, in Stolt migration this method was not used. Instead, a constant time shift was considered in order to compensate for the time delay inside the wall.

V. RECONSTRUCTION RESULTS AND ANALYSIS

In this section, we show and compare the reconstruction results of the imaging algorithms described in Section III for the aquarium. The absolute values of the focused images using SAR, Kirchhoff migration and Stolt migration are depicted in Fig. 4, 5 and 6, respectively. Fig. 7 shows some range-profiles of the focused SAR image.

From the figures we can see that the aquarium as well as the rear wall are visible in all migrated images. However, they are most clearly visible in the SAR result. SAR and Kirchhoff migration give almost the same location for the front wall (at approximately 58 cm far from the radar) and the aquarium (at approximately 1.78 m) while these estimated by Stolt migration are respectively 56 cm and 1.82 m. The rear wall is estimated at about 4.8 m far from the radar systems in all methods. Note that there is a shift of about 6–8 cm in the location of the front wall compared to the ground truth measured from the front size of the antennas to the wall.

TABLE I

RESULTS OF THE CONSIDERED IMAGING METHODS.

Method	Comp. complexity	SCR	RPE
SAR	$O(N_x N_z N_a)$	2.57	1.136%
Kirchhoff migration	$O(N_x N_z N_a)$	1.64	1.136%
Stolt migration	$O(N_x N_z \log_2 N_a)$	1.51	3.409%

The SCR and RTE of these methods for the aquarium are given in Table I. Note that the values of the SCR depend on

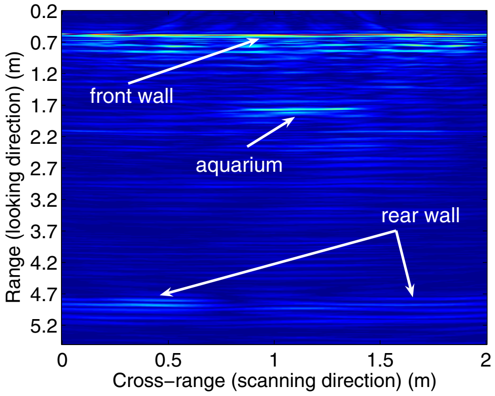


Fig. 4. Focused image using SAR.

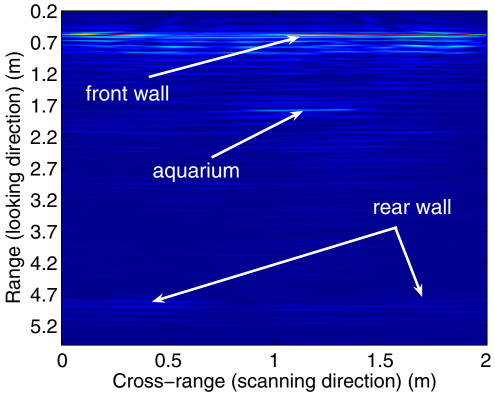


Fig. 5. Focused image using Kirchhoff migration.

the region in which it is calculated. However, we can conclude that SAR and Kirchhoff migration are of the same accuracy which is higher than that of Stolt migration. Besides, SAR gives the strongest object signal while Stolt migration gave the weakest one.

Concerning the computational complexity, Stolt migration is the fastest method, even compared with the fast SAR algorithm [9]. Indeed, if $N_x = N_z = N_a = N$, their computational complexity are respectively $N^2 \log_2 N$ and $N^{5/2}$.

VI. CONCLUSIONS

We have investigated the performance of three basic imaging methods (SAR, Kirchhoff migration and Stolt migration) in through-wall scenarios. The results showed that for the considered data, SAR is the best one in terms of accuracy and quality of the focused image. However, Stolt migration is the fastest method among them.

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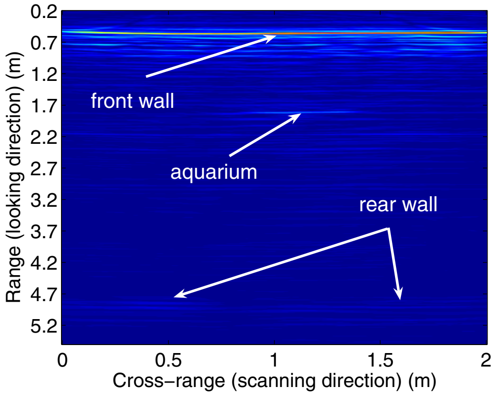

Fig. 6. Focused image using Stolt migration.

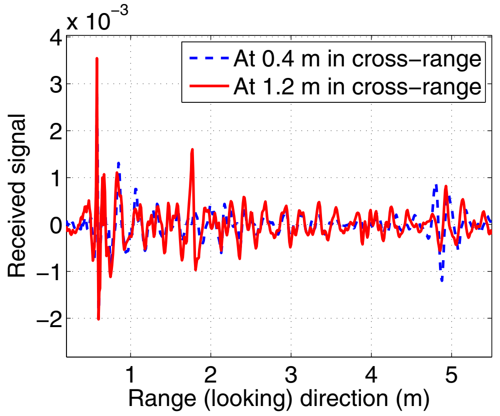

Fig. 7. Focused signals in the range direction using SAR.

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